Algebra

Goals for this chapter:

1. Lean the meanings of "equation" and "variable"
2. Lean how equations work
3. Learn to solve equations
4. Learn to solve simultaneous equations

All About Equations

Algebra is one of the most important math skills you will need for the GMAT. Almost every GMAT question [**NUMBERS? STATISTICS?**] requires some knowledge of algebra. In order to solve more difficult questions within the time limits algebra must be second nature – you must be able to focus your attention solely on the problem at hand, without having to try and remember how to do the underlying algebra.

Algebra is all about **solving equations**. <u>An **equation** is a comparison between two values</u>; this comparison must be exact. For example, telling you that x is 4 greater than y is an exact comparison between x and y, because it tells you the exact relationship between the two. On the other hand, saying x is greater than y is *not* an exact comparison, because we don't know how much greater, so it is not an equation.

Equations are written with equal signs "=". There must be numbers (or variables) on either side of the equal sign; "x =" is not an equation. The equal sign tells you that what is on one side of the sign is identical (equal to) what is on the other. Simple enough, right? That is why equations are comparisons – they tell you that two values (the left side and the right side) are equal to each other.

Let's look at a simple equation:

$$15 = 15$$

This is definitely true. If we change the way it's written, we get:

$$10 + 5 = 15$$

This is also true; if we do the math on the left side, it ends up being equal to the right side. 10 + 5 and 15 are *identical* – we can put (10 + 5) wherever we see a 15, and the results will always be the same. For example:

$$15 + 5 = 20$$

Replace 15 with (10 + 5)

$$10 + 5 + 5 = 20$$

Still true. <u>The two sides of an equation are **identical**, which means that you can replace everything on one side with everything on the other side, and the results will always be the same</u>.

Generally, equations contain **variables**; a variable stands for a number. Take a look at the following equation:

x + y = z

This means, "When you add the number x stands for to the number y stands for, you get the number z stands for," or "The sum of the numbers x and y stand for is identical to the number z stands for."

When you **solve** an equation, you find out what numbers each variable stands for; that is, you find out what number each variable is equal to. You want a result that looks like this:

x = [some number]

Notice that this result is going to be an equation. What happens is that we start with an equation (the equation we are trying to solve), and we move things around in it until we get a version of it that tells us what the variables are equal to. In reality, we never actually change the equation, or add anything new to it; we only change the way the equation looks. There is a good reason for this: the equation is the only information we have, so we have nothing we can add to it, and if we were to really change it, we wouldn't be solving *that* equation, we'd be solving a different one.

Changing Equations

We'll first explore the mechanics of how equations work, and how they can be re-arranged; later, we'll see how this applies to solving equations. The rule with equations is that whatever you do to one side you must always do the same thing to the other side. Why? Let's look back at a simple example:

15 = 15 True

Now, what if we change one side, say by adding 7 to it?

15 + 7 = 15 False, because it means
22 = 15 False!

Adding 7 to one side made the equation false, because the two sides are no longer equal. How can we made the sides equal? Add 7 to the other side:

22 = 15 + 7 True

The two sides start out equal; if we add to one side, the only way to keep them equal is to add the same amount to the other side. You can keep two equal amounts equal, as long as you always change them in the same way.

Let's play around with this. Take the following example:

10 + 5 = 15 True

Again, add 7 to one side:

10 + 5 = 22 False

We need to add seven to the other side. Does it matter where on the other side the 7 goes? Let's try putting the 7 in every possible position on the left side. Work along with me (I'll write it out, you do the arithmetic).

7 + 10 + 5 = 22	True
10 + 7 + 5 = 22	True
10 +5 + 7 = 22	True

It doesn't matter where we add a number to a side. The same holds for subtraction. If we take 7 away from one side and we get 8; we need to take 7 away from the other side to make them equal:

-7 + 10 + 5 = 8	True
10 – 7 + 5 = 8	True
10 + 5 – 7 = 8	True

What about multiplication? If we multiply one side by a number, we need to multiply the other side by the same amount, or they will no longer be equal. Here, if we multiply one side by 3 we get 45; we must multiply the other side by 3 as well.

3(15) = 45	True

Let's make it a little more complicated:

10 + 5 = 15	True
10 + 5 = 15(3)	False

Since we multiplied the right side by 3, we need to also multiply the left side by 3. What does this mean? Do we multiply the 10 by 3, the 5 by 3, or both? Let's try each and see what happens; again, I'll write it out, you do the arithmetic.

3(10) + 5 = 45	False
10 + 5(3) = 45	False
(3)10 + (3)5 = 45	True[1]

<u>When multiplying one side by some amount, we must multiply *every separate number* on that side by that amount</u>. There is a good reason for this. An equation tells us that the two sides are equal, not that any piece of one side is equal to the other side. If we change one side, we need to change the *entire* other side, not any one piece of it, to keep them equal. (I'll discuss what numbers are separate and which are not in a second; for now, any numbers that you are adding or subtracting are separate.)

The same is true for division. <u>When we have divided one side by some amount, we must divide *every separate number* on the other side by the same amount to keep them equal</u>. If we divide one side by 5, we get 3. Let's go through the options:

(10 ÷ 5) + 5 = 3	False

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[1] If you're still hazy on the arithmetic, let me explain why this works out. The **order of operations** (see Appendix **XXXX** on Arithmetic) is PEMDAS – Parenthesis, Exponents, Multiplication, Division, Addition, Subtraction; we do Multiplication before Addition and Subtraction. So we do the 3 times 10 and 3 times 5 before we add the two. 3 times 10 is 30, and 3 times 5 is 15, so we get 30 + 15, which is 45.

$10 + (5 \div 5) = 3$ False
$(10 \div 5) + (5 \div 5) = 3$ True

This last works because $10 \div 5$ is 2, and $5 \div 5$ is 1; 2 + 1 is 3, so we get 3 = 3. Thus, the equation only works when we divide every separate number on the side by 5.

We have seen how multiplication and division work when you have several numbers added together – you have to multiply and divide each number that is added together. What about when you have several numbers multiplied together? For example:

$3(5) = 15$ True

Now, if we multiply one side by 2, we have to multiply the other side by 2 also. But, do we multiply the 3 by 2, the 5 by 2, or both?

$(2)(3)(5) = 15(2)$ True
$(3)(2)(5) = 15(2)$ True
BUT
$(2)(3)(2)(5) = 15(2)$ False

Notice, that <u>when we have a group of numbers multiplied together, and we want to multiply them by some other number, we only multiply the whole group by that number, not every individual number in that group</u>. Why is this? This takes a bit of thinking about what these arithmetic operations mean. (See Appendix **XXXXX**, on Arithmetic)

3 times 5 means, "Add up three fives." Multiplying this by two means doing this twice – adding up three fives, and then adding three more fives to it. This is the same as 6 times 5 or 3 times 10 – 6 times 5 is "Add up six fives (or add up three fives and then three more)" and 3 times 10 is "Add up three tens (each ten being two fives, this is the same as adding three fives and then three more)."

On the other hand, take 10 + 5. What is two times this? Two times anything means "Add it up twice," so we have to add up 10 + 5 twice; this gives us two 10s and two 5s, the same as multiplying the 10 by 2 *and* the 5 by 2.

The same holds when dividing. The point is this: <u>treat groups of numbers multiplied together as one number – dividing or multiplying any piece of the group is the same as dividing or multiplying the whole group</u>. <u>With addition or subtraction, you must divide or multiply every term separately</u>.

Let's look at one more example to cement this.

$15 = 15$ True
$10 + 5 = 15$ True
$2(5) + 1(5) = 15$ True

Remember, 10 = 2(5); since the two are equal, I can replace any 10 I see with 2(5). Now, let's multiply both sides by 3.

$3(2(5) + 1(5)) = 3(15)$

Now, apply the rules we just went through. We want to multiply the left side by 3… we have two numbers added together, and each of those numbers is made up of two numbers multiplied together. What pieces do we multiply by 3? I'll give a second to work it out.

That's right, we want to multiply 2(5) by 3, and 1(5) by 3; each of these is treated as one number, so it gets multiplied, but we *don't* multiply the 2 and the 5 both by 3. So, we get

$$3(2)(5) + 3(1)(5) = 3(15) \quad \text{True!}$$

Solving Single Equations with One Variable

Let's say we wanted to solve the following equation for x:

Example 1: $x + 7 = 21$

Since we want to solve for x, we want to end up with what x equals; that is, we want x all by itself on one side of the equals sign, and everything else on the other (this "everything else" will be what x equals). Remember, the point is not to change the equation, but to re-arrange it. By not changing the equation, we don't change what x is, so whatever x ends up equaling in the end is the same thing x equaled all along.

To get x all by itself, we need to get rid of the 7 from the left side. The equation currently tells us what 7 more than x is; to find out what x is, we need to take 7 away from this. So, subtract 7 from both sides.

$$x + 7 - 7 = 21 - 7$$

$$x = 14$$

This should not be surprising; we get 21 when we add 7 to x, so x is 7 less than 21, which is 14. The interesting point is this: if we add something on one side, we get rid of it by subtracting it from both sides; if we know what more than x is, we find out what x is by making the sides smaller. This holds true for any operation: <u>in order to get rid of a number from one side of an equation, do the opposite operation (to both sides) from that which it is currently taking part in</u>. If you are multiplying by some number, divide by it; if you are subtracting, add; if you are dividing, multiply.

Let's apply that rule to this example:

Example 2: $3x = 36$

First, a note. "3x" is the same as "3(x)" or "Three times x." Here, then, we know what three times x is; to find out what x is, we chop this up into x sided pieces (which means we divide by 3 – we know that the number 3x is made up of three x's; if we divide it into three pieces, each will be an x). We divide both sides by 3:

$$\frac{3x}{3} = \frac{36}{3}$$

$$x = 12$$

Now an example with division:

Example 3: $\frac{a}{4} = 4$

Here, we know that one fourth of a is 4; to figure out what a is, we have to combine four of these fourths together (which is the same as multiplying by 4). Alternately, we simply follow our rule: the fraction sign always means division, and, as our rule says, to get rid of the division by 4, we multiply both sides by 4.

$$(4)\frac{a}{4} = 4\,(4)$$

$$a = 16$$

Now lets look at more complex equations, which will take more than one step to solve.

Example 4: $2x - 4 = 8$

Here, to solve for x we need to move the 2 and the 4 to the other side. Since the 2 is multiplied by x, we need to divide both sides by 2. Since the 4 is subtracted from x, we need to add 4 to both sides. Which do we do first? Does it matter?

Well, the order in which you move the numbers around generally doesn't matter in terms of getting the problem right: you can do the steps in any order (as long as you do them correctly) and get the right answer. However, it may be easier to do one way rather than another. When you are first practicing, try solving equations different ways (doing the steps in different orders) and see which is easier for you. Gradually, you'll come to be able to recognize the easier way when you first see a problem. I'll point out some rules of thumb as we go through the book.

Here, we'll try taking care of the 4 first. We have to add 4 to both sides.

$$2x - 4 + 4 = 8 + 4$$

$$2x = 12$$

Now get rid of the 2 by dividing both sides by 2.

$$\frac{2x}{2} = \frac{12}{2}$$

$$x = 6$$

Let's do it the other way (division, then addition) and see if it any easier or harder.

Example 4b: $2x - 4 = 8$

$$\frac{2x}{2} - \frac{4}{2} = \frac{8}{2}$$

Remember, dividing a side means dividing every separate number. A separate number is any number which is added or subtracted from another number. We have to divide the 2x, the 4 and the 8, because all are separate.

$$x - 2 = 4$$

$$x - 2 + 2 = 4 + 2$$

$$x = 6$$

Was that any harder? Not really… however, here is a generally good rule of thumb: it is typically easier to divide both sides after you do all other operations. This is because other operations, like addition and subtraction, will combine like terms. When you subtract or add numbers from both sides, it will end up causing numbers to combine; for example, in this question the 4 combined with the 8. Combining like terms will cause there to be less terms (once the 4 and the 8 combine, there is only one number, 12, remaining), which means we will have to do less division.

Try this example. Try and do it on your own before you read the explanation. If you see more than one method, try both.

Example 5: $6 + \frac{x}{3} = 15$

Alright, now here is the solution:

Solution for example 5:
We need to get rid of the 6 or the 3 first. Let's try the 6 first, since it needs to be subtracted from both sides; multiplication is similar to division in that you should do addition and subtraction first, because that will get rid of some terms and make multiplication easier.

$$6 - 6 + \frac{x}{3} = 15 - 6$$

$$\frac{x}{3} = 9$$

Because we are dividing x by 3, we get rid of the 3 by multiplying both sides.

$$(3)\frac{x}{3} = 9(3)$$

$$x = 27$$

One more example; again, do it on your own before looking at the solution:

Example 6: $4x + 4 = 18$

Solution:
We need to get rid of the two fours. We will have to divide by 4, and subtract by 4. Following our rule of thumb, we subtract by 4 first, because that will give us less division to do.

$$4x + 4 - 4 = 18 - 4$$

$$4x = 14$$

Since we have 4 times x, we have to divide both sides by 4.

$$\frac{4x}{4} = \frac{14}{4}$$

4 doesn't divide evenly into 14, so we simply reduce the fraction. Reducing a fraction means seeing if the same number can be divided into both the top and bottom of the fraction. If it can, we then divide both by it (see Appendix **XXXX** on Arithmetic for more on reducing fractions). Here, 14 and 4 can both be divided by 2, so the new fraction is what we get when we divide 14 by 2 and 4 by 2:

$$x = \frac{14}{4} = \frac{2(7)}{2(2)} = \frac{7}{2}$$

A Note on GMAT Technique

The key to successful algebra on the GMAT is to *never skip steps*. This means <u>always write every step down, never do anything in your head</u>.

Bad	**Good**
$2x - 7 = 5$	$2x - 7 = 5$
$2x = 12$	$2x - 7 + 7 = 5 + 7$
$x = 6$	$\frac{2x}{2} = \frac{12}{2}$
	$x = 6$

Notice the difference between the good and bad examples. Both people had to go through all the same steps (the person doing the bad example couldn't know that x = 6 unless they'd divided both sides by 2), but the good person wrote everything they did down. The bad person probably felt faster, because they did less writing… Why is skipping steps bad?

1. Doing work in your head requires more mental energy – you have to think more about what you are doing and you have to remember all the steps, rather than having the paper remember them for you. This tires you out in the long run.

2. As you get tired, the rate at which you can do work in your head slows dramatically, and the accuracy of this work drops as well. This is because your brain doesn't work as well when you are tired, so a method which relies on your brain is less effective. On the other hand, writing everything down relies much less on your brain, and much more on your habits. Because it doesn't use as much brain power, it doesn't become significantly less slower or less accurate as you tire.[2]

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[2] In fact, I've noticed, for myself, that I will often *speed up* at algebra problems as I get tired. This is because I'm less likely to think about what I do and more likely to just stick to a method that's become a habit for me; since I have good habits, the less my brain interferes with my algebra the faster I get.

3. When you make dumb mistakes (writing things down wrong, doing arithmetic wrong) , and you will, you are much more likely to catch them if you can see them written down in front of you.

<u>To summarize, writing every step down is easier, more reliable, and faster in the long run than doing work in your head</u>. It is a habit that you have to begin cultivating now; from now on, always write any step, any piece of math you do, down.

Practice Questions – Algebra Set 1

Do these practice questions before you go on to the next section. Solve each equation for x. An answer key can be found at the end of the chapter, and explanations can be found in the **XXXXXX** back of the book.

1. $2x + 8 = 42$

2. $4x - 3 = 61$

3. $3x = 2x - 7$

4. $\frac{x}{6} - 12 = 50$

5. $x + 3 = \frac{x}{2}$

6. $\frac{2x}{3} - 1 = \frac{x}{2} + 3$

Solving Single Equations with Multiple Variables

Equations often contain more than one variable.

Example 7: $x + y = 6$

If the GMAT wants you to solve for x, they'll typically ask, "Solve for x in terms of y." Don't be afraid when you see "in terms of"; it means only that our solution will include a variable (in this case a y). Solving for x in terms of y is just like solving for x – the answer will look like "x = something", but the "something" will include another variable. To solve for x, what do we do? That's right – we get the x on one side, and everything else on the other side.

Here, we have to move the y to the other side. Since we are adding y, we have to subtract it to move it to the other side. This is because y stands for some number; to move any number to the other side, we have to do the opposite operation.

$$x + y - y = 6 - y$$

$$x = 6 - y$$

Pretty easy, right? Let's do another example. Solve for x in terms of b.

Example 8: $bx = 6$

This example tells us that if we add up b number of x's, we'll get 6. To see what x is, we have to divide by b – this will split the number "bx" into x sized pieces (because we know that there are b number of x sized pieces in bx). Another way of looking at it is that our rule tells us that to get rid of b we have to do the opposite operation. Here we are multiplying by b, so we have to divide.

$$\frac{bx}{b} = \frac{6}{b}$$

$$x = \frac{6}{b}$$

Remember, when you divide bx by b, the b's will cancel out. When you divide a number by itself, the result is 1; thus, $b \div b = 1$, so $x(b) \div b = x(1) = x$. We know that $x = 6 \div b$, but, since we don't know what b is, we can't simplify this any further. We're done.

Practice Questions – Algebra Set 2

Solve each of these equations for x.

1. $y + 2x + 3 = x - 11$
2. $y = 3x + 6$
3. $\frac{x}{2} + 3 = 3y + 2$
4. $\frac{x}{y} + 2 = 27$
5. $2y + \frac{x}{2} = 4 + y$
6. $a\left(\frac{x + 3}{2}\right) = 7$
7. $5x - y = 3y + x$
8. $\frac{y}{3} - 4 = 3x + 12$

Solving Simultaneous Equations

The phrase "simultaneous equations" means you have multiple equations which talk about some (but not necessarily all) of the same variables. <u>If, in one question, x occurs in two equations, x is the same in both</u>.[3] This is not true from question to question – if x is 6 in question 8, it doesn't have to be 6 in any other question. Having simultaneous equations can help you in solving for multiple variables.

Let's say you knew

Example 9: $x = 6 - y$

You also knew that

$y = 2$

Here we have simultaneous equations – we have two equations which both talk about some of the same variables (y in this case). Can we figure out the value of x? If so, how?

We do this by putting all of our given information together. We know that y = 2. This means that 2 is *exactly the same* as y. Anywhere we see a y, we can put a 2.[4] For example

$y = 2$

becomes, if we replace y with 2,

$2 = 2$

This is true, but not very informative. Still, the fact that it is true means we did something right. However, what if we replace y with 2 in the *other* equation.

$x = 6 - y$

becomes

$x = 6 - 2$

$x = 4$

Ta da! Here, the combination of the information from one equation with the other equation helped us solve for x. Now, that wasn't too hard; one equation just *told* us what y was. The same process, however, will work in more complex equations as well.

Example 10: $a = 3b - 7$
$b = a + 1$

Solve for a.

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[3] We call them "simultaneous equations" because they are true *at the same time*, whereas equations from *different* questions are not simultaneously true – often they contradict each other.

[4] We could also put a y anywhere we see a 2, put this wouldn't be terribly helpful. We want to reduce everything to numbers, so we want to get rid of variables, not put more in.

Now, we know that b = a + 1, which means that b and (a + 1) are exactly the same. Anywhere we see "b" we can put in "a + 1" and vice versa. This is called **substitution**. Substitution is replacing a value with a value equal to it (generally, replacing a variable with something else). If we just focus on "b = a + 1", we can substitute "a + 1" for "b" and get

$$a + 1 = a + 1$$

Again, true, but not very informative. Notice that substituting information from an equation into itself never tells us anything useful. So, instead, we'll substitute into the *other* equation, replacing any b's with "a + 1".

$$a = 3(a + 1) - 7$$

I put the "a + 1" into parenthesis because we don't know how it interacts with the 3 yet. Do we multiply the a by 3, the 1 by 3, both, or neither? The "3b" in the original equation means "3 times b", which is the same as adding b + b + b. Well, b is the same as "a + 1", so b + b + b = a + 1 + a + 1 + a + 1, which is 3a + 3. *That* is the same as 3 times a + 3 times 1. Notice that we multiply every part of the a + 1 by 3. This should make sense, we want 3 b's, and b is a+1, so we need 3 of all the parts which make up b. The rule is, when you substitute for a variable, everything that was multiplied by the variable gets multiplied by every separate number (numbers added or subtracted from each other) you substitute in. This is similar to the rule for multiplying both sides of the equation, as we discussed above, and it should make sense, because numbers multiplied together are always treated as one number, where numbers added and subtracted together are treated separately.[5]

Let's finish this problem.

$$a = 3a + 3 - 7$$

$$a = 3a - 4$$

Move the a's to the same side. Since we have added 3a to one side, we have to subtract it from both sides.

$$a - 3a = 3a - 3a - 4$$

$$-2a = -4$$

$$\frac{-2a}{-2} = \frac{-4}{-2}$$

Don't forget to get rid of the negative sign. One of the common mistakes beginners make when solving is to not watch out for negative signs. If we want to solve for a, and we know what –a equals, we have to move the negative to the other side by either dividing or multiplying both sides by –1.

$$a = 2$$

[5] For example, if z = 3q, and we substituted that into the equation 3z + q = 4, we'd get 3(3q) + q = 4, which would turn into 9q + q = 4; we only multiply the 3 by one of the terms in 3q, because these are not separate numbers.

What, by the way, is b? It's easy to find out now that we know a; substitute the value of a into either equation and you'll get b.

$$b = a + 1$$

$$b = 2 + 1$$

$$b = 3$$

Notice that the two examples we've done so far have been very easy to substitute with, because we were told what one variable equaled. Consider a different case.

Example 11:

$$d + 2e = 10$$
$$3d - 3e = 12$$

Solve for d.

Before, we could substitute for one variable in the other equation because we knew what that variable equaled, so we could replace every occurrence of that variable with its equivalent. Here, we don't know what *either* variable equals. We can't substitute for e if we don't know what e is equal to, can we? No, we can't, so we have to first solve for one variable. <u>The first step in solving simultaneous equations is to always solve one equation for one variable</u>.

It doesn't actually matter which variable we solve for first; once we have gotten a value for one variable, we can always go back and solve for the other (as we saw in Example 10). Where *should* we start? There are two possible strategies:

Solve for the variable that looks easiest
Solve for the variable we *aren't* being asked about (here, e)

The first should make some sense; why would we do the 2nd? Well, if we solve for e, we then substitute that into the other equation and we'll be able to get a value for d (as we saw in both Example 9 and 10). This will be the answer to the question, and we'll be done. Thus, if we always start with the variable we aren't asked about, we'll be going more immediately for the answer. However, this isn't always the fastest way to solve the question; often, it's easier to solve for the easier variable. You'll see this as you get more experienced. Here, we'll try both ways, and then talk about the ins and outs later.

First, we'll solve the easiest looking equation. That's solving the 1st equation for d, because all we'll have to do is move the "2e" to the other side, whereas in the 2nd equation we'd have to move something to the other side and then divide. To move the 2e over, we subtract it from both sides.

$$d + 2e - 2e = 10 - 2e$$

$$d = 10 - 2e$$

Now we know what d equals. We can substitute "10 – 2e" anywhere we see a d. Remember, substitute into an equation other than the one we solved. This means substitute into the 2nd equation; replace every instance of d with "10 – 2e".

$$3(10 - 2e) - 3e = 12$$

Remember from Example 10 that when you substitute for a variable, anything that was multiplied by the variable gets multiplied by every separate number you substitute in. So, we multiply the 3 by the 10 and by the 2e. 3 times 10 is 30 and 3 times 2e is 6e.

$$30 - 6e - 3e = 12$$

$$30 - 9e = 12$$

To solve for e, move the 30 to the other side. Now, do we add or subtract the 30? Well, it's a positive 30, so it's really being added to –9e, so we do the opposite, and subtract it.

$$30 - 30 - 9e = 12 - 30$$

$$-9e = -18$$

Now we have to get rid of the –9e. *Don't forget about the negative sign*.

$$\frac{-9e}{-9} = \frac{-18}{-9}$$

A negative divided by a negative is a positive, so

$$e = 2$$

Now we know what e is, but we were supposed to solve for d. Substitute this value of e into either equation and solve for d. I like the 1st equation, because it's simpler. Replace very e with a 2:

$$d + 2(2) = 10$$

$$d + 4 = 10$$

$$d + 4 - 4 = 10 - 4$$

$$d = 6$$

Ok, now let's try it the other way – solving for the variable that we weren't asked for, and then substituting from there. This means we solve for e first. Here are the equations again.

$$d + 2e = 10$$
$$3d - 3e = 12$$

I'm going to solve the 2nd equation first, because I can see that it'll work out very nicely when I divide by 3. In algebra, you can do things in almost any order you want, as long as you follow the rules of algebra, and get the right answer, so don't spend a lot of time trying to figure out what the best possible thing to do is. On the other hand, if you see something that looks easier, do it first.

To solve the 2nd equation, I'm going to move the d to the other side and divide by 3. Since the d is positive, I have to subtract it.

$$3d - 3d - 3e = 12 - 3d$$

$$\frac{-3e}{-3} = \frac{12 - 3d}{-3}$$

Now, dividing the "12 – 3d" by −3 is the same as dividing each separate number on top by negative three.

$$e = \frac{12}{-3} - \frac{3d}{-3}$$

$$e = -4 - -d$$

I hope this last step isn't confusing. We have 3d divided by −3, which is −1d (3 ÷ 3 is 1, the d stays where it is, and we keep the negative sign because there is only one of them). However, we are *subtracting* this from 12, so it's minus minus 1d. This is the same as adding 1d.

$$e = -4 + d$$

Take this and substitute it into the other equation, replacing every e with "-4 + d".

$$d + 2(-4 + d) = 10$$

Remember, multiply the 2 by every separate term we substituted in.

$$d + -8 + 2d = 10$$

Combine the d's, then solve by moving the −8 to the other side (we have to add it, since it's negative).

$$3d - 8 = 10$$

$$3d - 8 + 8 = 10 + 8$$

$$3d = 18$$

$$\frac{3d}{3} = \frac{18}{3}$$

$$d = 6$$

Same answer, and the two methods seemed about equal in difficulty. They won't always be, as you'll learn from experience. The important point, though, is that you can do algebra in whatever order you want and get the same answer. That said, my rule of thumb is always solve the easy equations first.

So we've seen how to solve when we have two equations and two variables. Can we do it when we have more than that? The following example, by the way, is more complex than you should expect on the actual GMAT. But it's good practice, and illustrates my point nicely.

Example 12:

$$f + g + h = 9$$
$$2f - g = 3h$$

$h - 2f = -5 - 2g$

Solve for f.

I want to know what f is. This would be easy, if I knew what g and h were. I don't… but I can find out! How you ask? Well, how did we do it in the previous examples? We solved for g and h, and substituted into other equations. We'll do that here, too. The 2nd equation will be easy to solve for g, because we just have to move the 2f over.

$2f - 2f - g = 3h - 2f$

$-g = 3h - 2f$

$(-1)-g = (3h - 2f)(-1)$

$g = -3h + 2f$

Substitute this into the 1st equation (since that one is nice and simple).

$f + (-3h + 2f) + h = 9$

$f - 3h + 2f + h = 9$

Combine the f's and h's.

$3f - 2h = 9$

Now we need to get rid of that pesky h. Let's solve the 3rd equation for h (since it looks easy), and substitute that in.

$h - 2f = -5 - 2g$

$h - 2f + 2f = -5 - 2g + 2f$

$h = -5 - 2g + 2f$

Notice, what will happen if we substitute this in our "3f – 2h = 9" equation? The h will disappear, but we'll get an g in it instead. This is no good – we wanted to get rid of everything except f. How can we get rid of the g in this 3rd equation, before we substitute it into the 1st equation? The same way we got rid of the g in the 1st equation – substitute in the 2nd! How convenient! So, replace every g in this 3rd equation with "-3h + 2f".

$h = -5 - 2(-3h + 2f) + 2f$

Now, this is –2 times "-3h + 2f"; that is going to reverse the sign of everything in the parenthesis. Anytime you multiply parenthesis by a negative, you reverse the sign of every term in the parenthesis.

$h = -5 + 6h - 4f + 2f$

$h = -5 + 6h - 2f$

Combine the h's by moving the 6h to the other side (by subtracting).

$$h - 6h = -5 + 6h - 6h - 2f$$

$$-5h = -5 - 2f$$

$$\frac{-5h}{-5} = \frac{-5 - 2f}{-5}$$

Because we're dividing by a negative, I'm going to switch the sign of everything on top.

$$h = \frac{5 + 2f}{5}$$

OK, what will happen if we substitute *this* into the 1st equation? Well, we'll get rid of the h, and have only f's left. Splendid! Here is the equation we're substituting into:

$$3f - 2h = 9$$

$$3f - 2\left(\frac{5 + 2f}{5}\right) = 9$$

OK, now we just solve like normal (except this is a weird one…). Get rid of the 5 first, so we can combine like terms (the f's and the numbers). Since we are dividing by 5, if we multiply everything by 5, it'll disappear. Note, we have to multiply *everything* on both sides by 5 (including the "3f").

$$5(3f - 2\left(\frac{5 + 2f}{5}\right) = (9)5$$

$$15f - 2(5 + 2f) = 45$$

Now multiply the 2 by everything in parenthesis to get rid of it, and then we'll combine the f's. Remember, it's a negative 2, so we have to reverse the sign of everything in the parenthesis.

$$15f - 10 - 4f = 45$$

$$11f - 10 = 45$$

$$11f - 10 + 10 = 45 + 10$$

$$\frac{11f}{11} = \frac{55}{11}$$

$$f = 5$$

And we are done. One thing you'll notice, it's important to keep your work organized, or you'll never remember where to substitute into.

Alright, now that we've actually done the problem, let's go back and figure out what we actually did. We started with three equations with three different variables. We solved one equation (the 2nd one) for one of the variables and we substituted that into the other two. Having done that, we essentially had two equations with two variables in them, since substituting eliminated one of the variables. Whenever you substitute into an equation, the variable you substitute for disappears. Once we have two equations with two variables, we solve like normal – solve for one the variables, substitute that into the other equation, which will make the variable disappear, and leave us with one variable in the equation, which can easily solve for.

What if we had four equations, with four variables? Same thing, just one more step. We solve one equation, and substitute into the other three. That eliminates one variable. Now we solve another equation, and substitute that into the other two. That eliminates a second variable. Finally, we solve a third equation and substitute that into the last, which leaves us with one variable, and we solve for that.

As long as we have as many equations as we do different variables, we can always solve for any variable we want – we just keep solving and substituting until we are down to one equation with one variable. This is a useful piece of information: <u>if we have as many different equations as we do different variables, we can always get the number value of any variable we want</u>. This is going to come in very handy when we do data sufficiency.

Now, does this mean that every equation has to have the number of variables in it? No; as long as the total number of different variables is the same as the total number of equations, we can always solve, because we can always keep substituting.

Example 13:

$$2k + 3 = 5n + 1$$

$$k + \frac{m}{2} = 3n + 3$$

$$\frac{3n}{2} - m = -3 - 2n$$

Solve for m

Notice here that that 1st and 3rd equations only have two of the variables in them, and the middle equation has all three. None of the equations has the same group of variables in them. Nonetheless, we have three total variables (k, m and n) and three total equations. Our rule tells us that we should be able to get the value of m.

Go ahead and try and solve this on your own. Then read the solution below.

Solution for Example 13

We want to turn one of the equations into an equation with one variable in it. At first, the 1st and 3rd equations look perfect – each only has two variables in them, so they are almost there. However, if we substitute either equation into the other, we'll still end up with equations with two variables. Why? Because both equations have different variables from each other, so when we cancel out one variable, we'll be introducing a new one. Don't believe me? Try it out yourself.

OK, that out of the way, we'll just follow the procedure I outlined above; solve one equation and substitute into each of the others. First, solve the 1st equation for k (because it's simpler) and substitute that into both equations (you can't really substitute it into the 3rd equation, because it has no k in it, but that's fine).

$$2k + 3 = 5n + 1$$

$$2k + 3 - 3 = 5n + 1 - 3$$

$$2k = 5n - 2$$

$$\frac{2k}{2} = \frac{5n - 2}{2}$$

$$k = \frac{5n - 2}{2}$$

Now substitute into the 2nd equation.

$$\frac{5n - 2}{2} + \frac{m}{2} = 3n + 3$$

Now we need to get rid of the n's. Solve the 3rd equation for m (it's much easier to solve for m than n; on the other hand, if we solve for n, we can substitute in and get m more directly. My rule is always to do it the easy way, even when it's less direct; try it the other way and see for yourself).

$$\frac{3n}{2} - m = -3 - 2n$$

$$\frac{3n}{2} - \frac{3n}{2} - m = -3 - 2n - \frac{3n}{2}$$

$$-m = -3 - 2n - \frac{3n}{2}$$

Let's get a common denominator so we can combine the n's. The common denominator will be 2; we want to put 2n over 2. We do that by multiplying by 2n by 2 over 2 (which is the same as 1, so it doesn't change the equation) which will give us 4n over 2.

$$-m = -3 - 2n\left(\frac{2}{2}\right) - \frac{3n}{2}$$

$$-m = -3 - \frac{4n}{2} - \frac{3n}{2}$$

$$-m = -3 - \frac{7n}{2}$$

$$m = 3 + \frac{7n}{2}$$

Substitute this into the 2nd equation.

$$\frac{5n - 2}{2} + \frac{m}{2} = 3n + 3$$

$$\frac{5n-2}{2} + 3 + \frac{\frac{7n}{2}}{2} = 3n + 3$$

Now, we have a fraction in a fraction. How do we deal with that? Well, let's treat it like any other fraction. How do we get rid of the 2 on the bottom? By multiplying everything in the equation by 2.

$$2\left(\frac{5n-2}{2} + 3 + \frac{\frac{7n}{2}}{2}\right) = (3n + 3)2$$

Remember, we have to multiply *everything* by 2. The 2 will cancel out with the 2s in the denominator on the left side. This leaves us with

$$5n - 2 + 3 + \frac{7n}{2} = 6n + 6$$

$$5n + 1 + \frac{7n}{2} = 6n + 6$$

I want to get rid of that last fraction, so I'm going to multiply by 2 again.

$$2\left(5n + 1 + \frac{7n}{2}\right) = (6n + 6)2$$

$$10n + 2 + 7n = 12n + 12$$

Combine the n's and the numbers.

$$17n + 2 = 12n + 12$$

$$17n - 12n + 2 = 12n - 12n + 12$$

$$5n + 2 - 2 = 12 - 2$$

$$5n = 10$$

$$\frac{5n}{2} = \frac{10}{2}$$

$$n = 5$$

We want to solve for m, so plug this into an equation with both n and m in it (the 3rd will do).

$$\frac{3(2)}{2} - m = -3 - 2(2)$$

$$3 - m = -7$$

$$3 - 3 - m = -7 - 3$$

$$-m = -10$$

$$m = 10$$

To recap, any time the number of different equations you are given is equal to the number of variables, you can always solve for all the variables.

Practice Questions – Algebra Set 3

Find the value of both variables in each set of equations.

1. $x + y = 5$
   $y - x = 1$

2. $2a + b = 14$
   $2b - a = 8$

3. $2d - 5 = 2c - 1$
   $d - 2c = 3c - 2d - 4$

4. $.75e + .3f = e + 2$
   $\frac{1}{2}e + \frac{1}{2}f = f - e + 1$

5. $\frac{g + 1}{3} = 2(h - g)$
   $6h = 5g$

6. $j + 2k = 4$
   $4k = 2j - 6$

7. $\frac{2m}{n + 5} = 2$

   $\frac{3m}{2n} = 3$

8. $\frac{q}{4} + .2r = \frac{r}{2}$

   $\frac{q}{3} - .3r = 1$

Backsolving

There will come a time when you will be faced with a problem that you are unsure how to solve. This should be rare when doing straight algebra questions, as we have been doing, since there isn't much the GMAT can do to confuse you on these. More likely you'll see an algebra question that you know you can do, but not in the time given. What do you do? If you spend too much time on the question, that will hurt your GMAT score. Should you give up and go on, or is that a different way to answer questions.

If we were doing algebra in most settings, the answer would be no. But we're doing algebra on the GMAT, and that gives us an advantage over regular algebra-doers. What information do we have on the GMAT that the typical high-school math student doesn't normally have access to? That's right, the answers. We have the answer to the question staring us right in the face on every single problem. All we have to do is test out each of the answers we are given until we find the correct one. This technique is called **backsolving**.

How do we actually test the answers? Let's look at an example.

Example 14: If (4x + 2)(x + 8) = 18x, what is the value of x?

A. -5
B. -4
C. -3
D. -2
E. -1

I use this question because I haven't taught you to solve this sort of equation yet. There are several challenges – first, how do you multiply (4x + 2) by (x + 8)? Second, if you do so, how do you solve the result (which will have an x^2 in it? I'll discuss both these issues in the chapter on **Factoring and Exponents**. Even if you do know how to do a question like this, you should quickly realize that this specific question will take you a while; it would be nice if you have a faster and easier method. The point of backsolving is that <u>you backsolve whenever you don't know how to do a question any other way, or another approach to a question will take too much time</u>.

Each of the answers here, (a) through (e), is potentially a value for x. Why x? Because x is what we are being asked to solve for. How can we figure out if (a), for example, *actually is* x? Well, we use our old friend substitution. We substitute an answer into the equation for x, and see if everything works. This will make more sense when we do it. Quick summary: <u>The first step in backsolving is to see for what variable the answers will be substituted</u>. <u>Next, you substitute an answer into the given equations and see if they work</u>.

There are various backsolving strategies for deciding which answer to try first. One, for example, is to try the easiest answer first (I prefer this one). An even simpler method is to go through the answers in order, from (a) to (e). We'll do that one here, and I'll teach you others later. We start with (a), which means we substitute –5 for x.

A.

(4(-5) + 2)(-5 + 8) = 18(-5)

(-20 + 2)(3) = -80

(-18)(3) = -80

We can stop right here; I'm not sure what -18 times 3 exactly is, but I know it isn't -80 (for several reasons, simplest of which is that 8 times 3 ends in 4, not 0). I know, then, that this doesn't work. <u>An answer doesn't work if substituting it in gives you a false statement</u>, such as 2 = 3 or something of the sort. If you don't see yet that this won't work, do the rest of the math.

$$-54 = -80$$

This is false. This answer is wrong, so let's try the next one. Substitute −4 in for x.

B. $(4(-4) + 2)(-4 + 8) = 18(-4)$

$(-16 + 2)(4) = -72$

$(-14)(4) = -72$

Again, I know that this is false because −14 times 4 is going to end in a 6 (4 times 4 is 16), not in a 2. But, if you didn't see that, do the multiplication.

$$-56 = -72$$

This is not true, so we try the next answer. Substitute −3 for x.

C. $(4(-3) + 2)(-3 + 8) = 18(-3)$

$(-12 + 2)(5) = -54$

$(-10)5 = -54$

$-50 = -54$

False, so this is also the wrong answer. I did the multiplication here because it was so easy. Now we substitute in answer D. If D doesn't work, we *know* E is the answer, and don't even have to try it out.

E. $(4(-2) + 2)(-2 + 8) = 18(-2)$

$(-8 + 2)(6) = -36$

$(-6)(6) = -36$

$-36 = -36$

Aha! This is true! That means that this is the answer.

Let's talk a little about how this works. First, notice that backsolving transforms an algebra problem into an arithmetic problem. When the algebra is too hard, this will make it much easier. Does this mean that you should backsolve all the time? No, because most of the time doing the algebra will be faster than backsolving, because you'll be good at algebra, and because backsolving often requires looking at several answers. That said, backsolving is a *crucial* technique to have in your repertoire; you *must* be able to deal with questions that you can't answer normally, since you are guaranteed to come across them, and this is what backsolving is for.

> **Backsolving Story**
> A tragedy
>
> I once had a student. She had signed up with me after completing a course at one of the large test preparation companies. On a recent diagnostic test, she had scored at 670, but she wanted to do better and felt that the company couldn't help her any further. We worked together and I because fairly frustrated. She was very smart and was capable of scoring at least a 750, but she *refused* to learn to backsolve. I think her problem was that she was so smart; she was used to solving problems head on, and backsolving a question was like admitting that she couldn't do it. Several times I had long talks with her, where I explained the importance of backsolving, how to do it, and when to do it. I gave her homework where she was required to backsolve, exercises to force her to get used to backsolving, and so forth, but she would not do them. We'd meet the next week and she would have the same problems.
>
> So she went and took the GMAT. I called her after the test and asked how she did. To kill the suspense, she got a 710, a great score, but not as well as she could have done. In telling me about the test, she uttered the following sentence, "I got stuck on the first question! I didn't know how to do it, and I spent almost five minutes trying to figure it out. It threw me off for a lot of the test." I asked her, "Did you try to backsolve it?" but I already knew the answer. "No…" she said. If she had backsolved, she would have gotten the answer right, and done it in under two minutes, and she would have been in better mental shape for the remainder of the questions. I'm sure she would have gotten a much better score.
>
> The moral of the story is this: your goal is to get a good GMAT score. Use the techniques that get you that score, regardless of how they make you feel. You *will* come across questions that you can't do, or can't do in the time you are given. Backsolve these questions.

The summary of backsolving:

- Backsolve when you don't know how to do the question, or doing it "normally" will take too long
- To backsolve, substitute each answer into the equations for the variable you are being asked about in the question
- If an answer gives you a false statement, such as 1 = 2, it is wrong
- If an answer gives you all true statements (e.g. 2 = 2) it is right
- You don't have to do all the math for each answer, just enough to see that it will be wrong

Let's try doing one more example with backsolving.

Example 15:

$$\frac{x(11-x)+2}{\frac{4x}{3}+9} = \frac{4+\frac{30}{x}}{5x-8}$$

What is the value of x?

A. 1

B. 2
C. 3
D. 4
E. 5

Here we could definitely solve just by doing algebra: we only have one variable, and one equation. However, doing so would be tremendously complicated and time consuming – given all the complex fractions there would be a huge amount of multiplying by denominators, which would result in large mess of an equation. You might try doing it algebraically just to see how difficult it is. Backsolving will definitely save us some time.

Let's start with A. Substitute 1 in for x.

A.

$$\frac{1(11-1)+2}{\frac{4(1)}{3}+9} = \frac{4+\frac{30}{1}}{5(1)-8}$$

$$\frac{1(10)+2}{\frac{4}{3}+9} = \frac{4+30}{5-8}$$

$$\frac{12}{\frac{4}{3}+9} = \frac{34}{-3}$$

I will do no more math. The right side is negative, because it's a positive divided by a negative number, and the left side is positive, because it has all positives. A positive can't equal a negative, so we try answer B.

B.

$$\frac{2(11-2)+2}{\frac{4(2)}{3}+9} = \frac{4+\frac{30}{2}}{5(2)-8}$$

Remember that a fraction sign means "divide", so 30/2 is 30 ÷ 2, which is 15.

$$\frac{2(9)+2}{\frac{8}{3}+9} = \frac{4+15}{10-8}$$

$$\frac{20}{\frac{8}{3}+9} = \frac{19}{2}$$

Ok, this isn't so obvious, so I have to do the math some more, or use a little trick. The trick? Skip this answer, and hope that the other ones are more obvious. But, since I'm teaching here, we might as well do the math. Get a common denominator on the bottom of the left fraction. The denominator will be 3, so multiply 9 by 3/3.

$$\frac{20}{\frac{8}{3} + 9\left(\frac{3}{3}\right)} = \frac{19}{2}$$

$$\frac{20}{\frac{8}{3} + \frac{27}{3}} = \frac{19}{2}$$

$$\frac{20}{\frac{35}{3}} = \frac{19}{2}$$

How do we handle the complex fraction on the left side? Well, remember that the fraction line means "divide", so we really have 20 ÷ 35/3. When we divide by a fraction, we invert the 2^{nd} fraction and multiply. So, 20 ÷ 35/3 = 20 (3/35), which is 60/35. Re-read this paragraph and put it into your notes; you'll see complex fractions in the future, and you want to know how to handle them.

$$\frac{60}{35} = \frac{19}{2}$$

Reduce the fraction on the left. 5 goes into both numbers.

$$\frac{12}{5} = \frac{19}{2}$$

I know these are not equal. 5 goes into 12 about 2 times, where 2 goes into 19 about 8 times. The two aren't equal, so this is false. Try C.

C.

$$\frac{3(11 - 3) + 2}{\frac{4(3)}{3} + 9} = \frac{4 + \frac{30}{3}}{5(3) - 8}$$

Notice that a lot of the fractions cancel here. 30/3 is 10; with 4(3) over 3, the 3's cancel out, leaving just 4.

$$\frac{3(8) + 2}{4 + 9} = \frac{4 + 10}{15 - 8}$$

$$\frac{24 + 2}{13} = \frac{14}{7}$$

$$\frac{26}{13} = \frac{14}{7}$$

$$2 = 2$$

We're done.

Practice Questions – Algebra Set 4

Backsolve each of the following questions.

1. Solve for x.

$$\frac{\frac{x}{16} + \frac{x}{8} + \frac{x}{2}}{x - 5} = 1$$

A. 8
B. 11
C. 16
D. 24
E. 32

2. Solve for x.

$$\frac{x - 5}{x - 2} = .2x - 1$$

A. 1
B. 3
C. 4
D. 5
E. 10

3. $ab = 3 + 2a + b$

$a - b = -5$

What is the value of a?

A. .5
B. 2
C. 3
D. $\frac{10}{3}$
E. 5

4. Solve for b.

$$\frac{a + b}{ab} = \frac{2}{3}$$

$$a - b = \frac{a}{b} - 1$$

A. 1
B. 2
C. 3
D. 4
E. 6

Data Sufficiency

Up to this point we've talked only about **Problem Solving** type questions – those where you are asked to actually get a numeric answer to a question. The GMAT, of course, has another type of math question, **Data Sufficiency**. These questions are a whole different ball of wax. See the **About the Test XXXXX** chapter for more detail on Data Sufficiency; here's a short re-cap.

Data Sufficiency questions ask you not to *solve* the problem presented, but only if you *can* solve it. This means we don't necessarily have to do all the math, but only have to do enough to realize if we can finish it or not. However, be careful. Just because you don't have to solve the problem doesn't mean that you don't have to do any work. There are two common mistakes people make on Data Sufficiency. The first is solving the problem when they don't have to; this is a waste of time. The second, though, is trying to do everything in their heads; this leads to mistakes, just as it would in a Problem Solving question. <u>The key to Data Sufficiency is to write everything down, and use your algebra skills, but stop when you realize that a problem is either definitely solvable or definitely unsolvable</u>.

Algebra Data Sufficiency questions are likely to ask you for the **value** of a variable. This means that they what to know what number the variable equals. <u>If you can find a *single* number that the variable equals, that will be sufficient</u>. If you find that the variable can equal more than one number, it is insufficient, because you don't really know the value.

It's easier to learn Data Sufficiency by doing some example. We'll just run through a bunch of examples to illustrate some of the key concepts, especially how the answers work. One thing to know is that the answers are the same for every Data Sufficiency question.

Example 16: $x + y = 4$. What is the value of x?

1) $x > 2$
2) $y = 1$

A. Statement (1), by itself, is sufficient to answer the question, but statement (2), by itself, is not.
B. Statement (2), by itself, is sufficient to answer the question, but statement (1), by itself, is not.
C. Statements (1) and (2) taken together are sufficient to answer the question, although neither statement by itself is sufficient.
D. Either statement by itself is sufficient to answer the question.
E. Statements (1) and (2) taken together are not sufficient to answer the question, nor are they sufficient to answer the question by themselves.

<u>In data sufficiency, we look at each statement (1 and 2) by themselves, ignoring the other one</u>; this means that information (such as equations) given in one statement can't be used when considering the other statement. <u>If both statements are insufficient, we consider them simultaneously</u>.

If $x > 1$, it could be 2 or 3 or 4 or any of a zillion other numbers. Since you can't get a single value for x, Statement 1 is insufficient. Make a note of this and move on. <u>Once you know that a statement is sufficient or insufficient, write it down</u>; this way, if the next statement is difficult, and you forget what you figured out for the previous statement, you don't have to re-do the work. Look at Statement 2 now (ignoring Statement 1)

If we know what y is, we can plug this number in to the equation "x + y = 4" and solve for x. This is sufficient. You don't have to put the statements together.

Since statement 2 is sufficient by itself, and 1 is not, the answer is B.

Example 17: x + y = 4. What is the value of x?

1) 2x + 4y = 11
2) x - y = 1

A. Statement (1), by itself, is sufficient to answer the question, but statement (2), by itself, is not.
B. Statement (2), by itself, is sufficient to answer the question, but statement (1), by itself, is not.
C. Statements (1) and (2) taken together are sufficient to answer the question, although neither statement by itself is sufficient.
D. Either statement by itself is sufficient to answer the question.
E. Statements (1) and (2) taken together are not sufficient to answer the question, nor are they sufficient to answer the question by themselves.

Always write down what you are told before you look at the statements. Here, you are told

$$x + y = 4$$

Now look at Statement 1, and write what you are told here down as well. <u>When you do Data Sufficiency, always label your work as belonging Statement 1 or 2</u>. We do this so that we don't think we have more information than we actually do, which might lead to wrong answers.

1) $2x + 4y = 11$

Now, if this was a Problem Solving question, could you solve for x, given these two equations? What is our rule?

That's right, when you have as many different equations as you do different variables, you can always solve for any variable. Here, we have two different equation (x + y = 4 and 2x + 4y = 11) and two different variables (x and y).[6] Thus we can solve, and this is sufficient.

Now we consider Statement 2. Again, we start off by writing the equation we are given down.

2) $x - y = 1$

We can still use the x + y = 4 equation, but we can't use the 2x + 4y = 11 equation. The first equation is given to us at the beginning, so it gets used for all the statements; the other equation can only be used when we look at Statement 1.

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[6] Just to remind you, you'd have to solve one equations for either x or y and then substitute that into the other equation, which would turn it into an equation with only one variable, which you could then solve. Here, we'd probably solve the 1st equation for y, since that's pretty simple, and plug that into the 2nd equation.

Here we have a second equation. Together with the x + y = 4 equation, we have two equations and two variables. Again, this is sufficient. Because both statements are sufficient on their own, the answer is D.

Example 18: What is the value of x?

1) $2x - y = 19$
2) $x + 3y = 7$

A. Statement (1), by itself, is sufficient to answer the question, but statement (2), by itself, is not.
B. Statement (2), by itself, is sufficient to answer the question, but statement (1), by itself, is not.
C. Statements (1) and (2) taken together are sufficient to answer the question, although neither statement by itself is sufficient.
D. Either statement by itself is sufficient to answer the question.
E. Statements (1) and (2) taken together are not sufficient to answer the question, nor are they sufficient to answer the question by themselves.

Here, unlike the previous examples, we are not given any information to start with. Go straight into Statement 1.

We have a single equation with two variables. There is no way to get a value for x (that is, to find out what number x equals), so this is insufficient.

Statement 2 is the same as above: we have one equation with two variables. This is insufficient as well. <u>When both statements are insufficient, you must consider them together</u>. This means that you can use all information (such as equations) given in either statement. Here, we are given one equation in Statement 1, and another equation in Statement 2. This gives us a total of two equations, and we have two variables (x and y). Following our rule, this must be sufficient, since we have an equal number of variables and equations. Since neither statement is sufficient on their own, but they *are* sufficient when put together, the answer is C.

Notice the process of Data Sufficiency: look at Statement 1 by itself; look at Statement 2 by itself; if one or the other is sufficient, pick an answer. If neither is sufficient, look at them together. Memorize this process and get used to it. You don't want to have to think about how you are supposed to do these questions on the GMAT; rather, you want to focus you attention on the questions themselves. Also, memorize the answers. The answers to every data sufficiency question are the same. Once you have them memorized, you will know what the answer is without having to re-read and puzzle them out.

Example 19: What is the value of x?

1) $2x + y = 14$
2) $x + y + z = 12$

A. Statement (1), by itself, is sufficient to answer the question, but statement (2), by itself, is not.
B. Statement (2), by itself, is sufficient to answer the question, but statement (1), by itself, is not.
C. Statements (1) and (2) taken together are sufficient to answer the question, although neither statement by itself is sufficient.

D. Either statement by itself is sufficient to answer the question.
E. Statements (1) and (2) taken together are not sufficient to answer the question, nor are they sufficient to answer the question by themselves.

Again, we are given no information to start with. Looking at Statement 1, and writing it down (remembering to label it as Statement 1)

1) $2x + y = 14$

we see that we have one equation with two different variables. We can't get a value for x, so this is insufficient. Jot that down and look at statement 2.

2) $x + y + z = 12$

This is also a single equation, with *three* variables. We definitely can't get a value of x. Since both statements are insufficient, we must consider them together. Together we have two equations with three variables. We won't be able to solve for x; if we substitute one equation into the other, the result will always have two variables in it. If you don't believe me, you're welcome to try and solve it. I'll wait…

OK, we see that, even combining the two statements, they are insufficient. They are also insufficient on their own. What answer is that? That's right, answer E. Excellent.

We're going to do two more examples, to show the sort of tricks the GMAT can pull on you.

Example 20 $x + 3y = 7$. What is the value of x?

1) $3x + 9y = 21$
2) $2x + 2y = 12$

A. Statement (1), by itself, is sufficient to answer the question, but statement (2), by itself, is not.
B. Statement (2), by itself, is sufficient to answer the question, but statement (1), by itself, is not.
C. Statements (1) and (2) taken together are sufficient to answer the question, although neither statement by itself is sufficient.
D. Either statement by itself is sufficient to answer the question.
E. Statements (1) and (2) taken together are not sufficient to answer the question, nor are they sufficient to answer the question by themselves.

Is Statement 1 sufficient by itself? It should look like it – you have two variables and two equations, right? Look a little closer, though. Here are your two equations:

$$x + 3y = 7$$
$$3x + 9y = 21$$

The first equation has x, the second 3x. The first has 3y, the second 9y. The first has 7, the second has 21. Notice a pattern? Every term in the second equation is three times something in the first equation. We can get the second equation by multiplying the first equation by 3 (or we could get the first equation by dividing the second equation by three). Is the second equation really a different equation from the first? No – it is just a different way of saying the first equation. For example, if I told you that a = 3, and then that 3a =

9, these are two ways of saying the *same thing*. Our rule is that you have to have as many different equations as different variables. You don't, so this is insufficient.

So, you don't believe me that this is insufficient? Let's try solving these two equations.

$$x + 3y - 3y = 7 - 3y$$

$$x = 7 - 3y$$

Substitute that into the other equation:

$$3(7 - 3y) + 9y = 21$$

$$21 - 9y + 9y = 21$$

$$21 = 21$$

True, but not very informative. If you look back at **Example 9** and **Example 10** you'll see that what we get here is exactly what we would get if we solved an equation and substituted it into itself. Substituting an equation into itself will never give you a value for any variable, because all the variables will disappear. There's a reason for this (which you don't have to understand, but I'll tell you anyway and you can ignore it if you'd like): when you solve an equation, you move all the other variables to one side by using the opposite operation (such as subtraction to move positive numbers over). When you substitute this back into the same equation, the variables you substitute in will each have a matching, but opposite, variable to contend with, and will all cancel out. The numbers will cancel out, too, if you do enough math – in the above example, if we had subtracted 21 from both sides we would have gotten 0 = 0.

The point is, having two of the same equation is never going to help you get a value for a variable.

Here's the trick to watch out for: <u>look out for one equation which is just a multiple of another equation (meaning that the 1st equation is just the 2nd equation times some number)</u>. When you have equations which are just multiples of each other, they only count as one equation.

How about Statement 2. Is this equation a multiple of the original equation? No, there is no number which you could multiply either equation by to make it look like the other. *Now* you have two variables and two equations, so this is sufficient. The answer is B.

The first time I ever looked at a GMAT problem was when I took a GMAT diagnostic test, back in the old days, when I was teaching for a large test-prep company. I wanted to start teaching GMAT, and we had to score above a certain level in order to do so. I had taught the LSAT and GRE, so I figured the GMAT would be a snap, and I didn't really study for it (other than briefly going over the answers to Data Sufficiency questions). The first question I got on the math section was a Data Sufficiency question; it was essentially Example 20, just with different numbers. I missed it, and ended up scoring a 720 on the test. (This, by the way, was good enough for me to teach the class, but didn't satisfy me, so I took another diagnostic, after actually studying, and scored much higher).

What's the point of the story? Well, there are several. First, no matter how smart you are (and I'll be honest and admit that I'm pretty smart), these tricks will catch you out if you aren't ready for them. The moral? Be ready for what the GMAT is going to test you on.

Second, *anyone* can miss questions when they study, and everyone does. Don't get discouraged if you don't do as well as you like when you are studying. Figure out what you are doing wrong, and correct it. It's OK to make mistakes when you study. It's *not* OK to not learn from those mistakes.

Third, you can still score pretty high, even if you miss the first question on the test, as I did. Most people really over-emphasize the first few questions on the GMAT, since, individually, they are worth more than any other question. Remember, the test is a test of how you do overall, not on any one question. You can miss any question you want, and still get a good score.

There is another way this trick could be played. You might have an equation which is not a multiple of another, but just a re-arrangement of the other. For example

$$4a + b = 16$$
$$3a + b = 16 - a$$
$$4a - 16 = -b$$
$$2a - 16 + 2b = -2a + b$$

Each of these equation is a re-arrangement of the others. If you move the variables around enough, you'll get the other equations. Thus, none of these (except the first) counts as a new equation. Don't worry about this overmuch, because it doesn't happen very frequently on the test, but keep your eye out for it.

Here's another interesting twist on Data Sufficiency. The concept illustrated by the next question will be explored in much more detail in later chapters, but it's worth thinking about now.

Example 21 If $x + y = z$, what is the value of z?

1) $x = 3$
2) $x + y = 2$

A. Statement (1), by itself, is sufficient to answer the question, but statement (2), by itself, is not.
B. Statement (2), by itself, is sufficient to answer the question, but statement (1), by itself, is not.
C. Statements (1) and (2) taken together are sufficient to answer the question, although neither statement by itself is sufficient.
D. Either statement by itself is sufficient to answer the question.
E. Statements (1) and (2) taken together are not sufficient to answer the question, nor are they sufficient to answer the question by themselves.

Statement 1 is clearly insufficient; if $x + y = z$, and $x = 3$, we know that $3 + y = z$, but we still don't know what number z is equal to. What about Statement 2? Take a second to think about it.

Most people are inclined to say that it is insufficient. After all, we have three different variables (x, y, and z) but only two equations. However, notice that we are looking for z, and z is equal to $x + y$. At the same time, $x + y$ are equal to 2. So, if z is the same as $x + y$, and *that* is the same as 2, then z is the same as 2, right? In other words, we can substitute 2 for (x + y) in the original equation, and that will give us the value of z.[7] Thus, Statement 2 is sufficient, although 1 isn't, so the answer is B.

Doesn't this break our rule about the number of variable and the number of equations? Why am I teaching you rules that don't really work? Darn me! But wait, calm down and let me explain. Look back at the rule as I stated it, on page **XXXX**. I said that any time we have the same number of variables as equations, we can solve. Does this mean that we can *only* solve when we have the same number of variables and equations? No, it doesn't. The rule still applies, this is just a situation that the rule doesn't cover.

So, here's an additional rule. <u>If you have more variables than equations, do a little math (i.e., solve and substitute), because you might be able to get a value for some of the variables</u>. Notice, though, that here you could not solve for x or y, only z. If we did have three equations, we could solve for any of the variables.

Here's the procedure for using this new rule. <u>First, count the variables and equations (watching out for equations which are really the same, as described above)</u>. <u>If the number of variables is the same as the number of equations, you can get a value for any variable</u>. <u>If not, do a little math (solve and substitute) and see if you can answer the question that way</u>. Memorize this process, practice it, love it, and Data Sufficiency questions will soon be an old hat.

Practice Questions – Algebra Set 5

1. What is the value of x?

    1) $x = 4 + y$
    2) $2x - 8 = 2y$

    A. Statement (1), by itself, is sufficient to answer the question, but statement (2), by itself, is not.
    B. Statement (2), by itself, is sufficient to answer the question, but statement (1), by itself, is not.
    C. Statements (1) and (2) taken together are sufficient to answer the question, although neither statement by itself is sufficient.
    D. Either statement by itself is sufficient to answer the question.
    E. Statements (1) and (2) taken together are not sufficient to answer the question, nor are they sufficient to answer the question by themselves.

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[7] Alternately, we could solve the 2nd equation and substitute. It doesn't matter which variable you solve for.

$$x + y - y = 2 - y$$

$$x = 2 - y$$

Substitute this into the original equation (x + y = z), replacing x with (2 – y).

$$2 - y + y = z$$

$$2 = z$$

2. If $x + 3 = 7y$ What is the value of x?

   1) $4x = 8y + 8$
   2) $x - y = 3y$

   A. Statement (1), by itself, is sufficient to answer the question, but statement (2), by itself, is not.
   B. Statement (2), by itself, is sufficient to answer the question, but statement (1), by itself, is not.
   C. Statements (1) and (2) taken together are sufficient to answer the question, although neither statement by itself is sufficient.
   D. Either statement by itself is sufficient to answer the question.
   E. Statements (1) and (2) taken together are not sufficient to answer the question, nor are they sufficient to answer the question by themselves.

3. $3x + 2y = z + 6$. What is the value of x?

   1) $y = .5z$
   2) $z - 2y = 0$

   A. Statement (1), by itself, is sufficient to answer the question, but statement (2), by itself, is not.
   B. Statement (2), by itself, is sufficient to answer the question, but statement (1), by itself, is not.
   C. Statements (1) and (2) taken together are sufficient to answer the question, although neither statement by itself is sufficient.
   D. Either statement by itself is sufficient to answer the question.
   E. Statements (1) and (2) taken together are not sufficient to answer the question, nor are they sufficient to answer the question by themselves.

4. What is the value of x?

   1) $2x + 3y = 7$
   2) $4x - y = 7$

   A. Statement (1), by itself, is sufficient to answer the question, but statement (2), by itself, is not.
   B. Statement (2), by itself, is sufficient to answer the question, but statement (1), by itself, is not.
   C. Statements (1) and (2) taken together are sufficient to answer the question, although neither statement by itself is sufficient.
   D. Either statement by itself is sufficient to answer the question.
   E. Statements (1) and (2) taken together are not sufficient to answer the question, nor are they sufficient to answer the question by themselves.

Practice Questions Answer Key

Algebra Set 1

1. $x = 17$
2. $x = 16$
3. $x = -7$
4. $x = 9$
5. $x = -6$
6. $x = 24$

Algebra Set 2

1. $x = -y - 4$
2. $x = \frac{y - 6}{3}$
3. $x = 6y - 2$
4. $x = 25y$
5. $x = 8 - 2y$
6. $x = \frac{14}{y} - 3$
7. $x = y$
8. $x = \frac{y}{9} - \frac{16}{3}$

Algebra Set 3

1. $x = 2$; $y = 3$
2. $a = 4$; $b = 6$
3. $c = 5$; $d = 7$
4. $e = 4$; $f = 10$
5. $g = -\frac{1}{2}$; $h = \frac{5}{12}$
6. $k = \frac{1}{4}$; $j = \frac{7}{2}$
7. $m = 10$; $n = 5$
8. $q = 12$; $r = 10$

Algebra Set 4

1. C
2. D
3. B
5. A

Algebra Set 5

1. E
2. D
3. B
4. C